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CONTACT PROBLEM FOR AN ORTHOTROPIC PLATE WITH A CIRCULAR HOLE LOADED BY A BOLT

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ABSTRACT

The contact problem associated with a bolt bearing on a circular frictionless hole in a rectangular orthotropic plate has been examined. The problem has been formulated by the well-known methods of Muskhelishvili and Lekhnitskii using functions of complex variables. The orthotropic stress functions are expanded in Laurent series and boundary conditions satisfied by boundary collocation. A simple iterative process has been utilized with success to find the contact angle. The process utilizes the physical fact that the radial stress in the contact region is compressive and reduces to zero in a well-behaved manner at the point where the bolt begins to separate from the plate. The problem was motivated by the growing need in composite designs to gain analytical understanding of the joint. Some interesting results concerning the stress concentrations occurring in plates with various geometries and material parameters are presented. The results show the importance of complex interactions and the necessity of employing the particular method to estimate stress concentrations in problems of this nature.

NOMENCLATURE

$a_{ij}(i,j = 1,2,6)$	Effective compliance coefficients of orthotropic laminates
D	Fastener diameter
E_1, E_2	Young's moduli of individual layers of laminate along material axes
e	Edge distance, pin center to free edge lying in the direction of applied load
G_{12}	Shear moduli of individual layers of laminate referred to material axis system
K_{NS}	$\sigma_x)_{90}/\bar{\sigma}_{NS}$, $K_{so} = \tau_{xy})_{max}/\bar{\tau}_{so}$, $K_B = \sigma_B)_{max}/\bar{\sigma}_B$
	Stress concentration factors for net section, shearout, and bearing stresses
K'_{NS}	$\sigma_x)_{90}/\sigma_A$, $K_{so} = \tau_{xy})_{max}/\sigma_A$, $K_B = \sigma_B)_{max}/\sigma_A$
	Ratios of peak net section, shearout, and bearing stresses to applied tension in laminate
P	Pin load
P_R, q_K	Coefficients relating displacement to Airy stress function
R	Pin radius
r	Radial distance from center of pin
s	"Side" distance, i.e., lateral distance from pin to free edge (single pin case) or symmetry line (multipin case) in direction normal to load
$s_k(k = 1,2)$	Coefficients in expressions for complex characteristics, z_k
t	Laminate thickness
U	Airy stress function
$U_k(k = 1,2)$	Complex components of Airy stress function
u, v	Displacements in rectangular coordinate system
x, y	Rectangular coordinates
$z_k(k = 1,2) = x + s_k y$	Complex characteristics of partial differential equation expressing compatibility in terms of U
α'_k, $\beta'_k(k = 1,2)$	Real and imaginary parts of s_k
α_n, β_n, γ_n, δ	Coefficients in Laurent series expansion for ϕ_1 and ϕ_2
$\zeta_k(k = 1,2)$	Conformal mapping functions with arguments s_k
η	Angle of contact at fastener-hole boundary
θ	Angular polar coordinate
σ_A	Uniform tension applied to loaded edge of plate
$\bar{\sigma}_B = (2s/D)\sigma_A$	Mean bearing stress
$\bar{\sigma}_{ns} = \left[\dfrac{2s}{D/(1 - \frac{2s}{D})}\right]\sigma_A$	Mean net section stress
$\sigma_B)_{max}$	Peak bearing stress (i.e., peak compression) at hole
$\sigma_x)_{90}$	Axial stress at $\theta = 90°$, $r = R$, usually equal to peak net section tension
$\tau_{so} = (s/e)\sigma_A$	Mean shearout stress
$\tau_{xy})_{max}$	Maximum shear stress along hole ($r = R$, usually equivalent to maximum shear stress in the laminate)
$\omega_k(k = 1,2)$	Inverse mapping function describing z_k as a function of ζ_k
ν_{12}, ν_{21}	Poisson's ratio

INTRODUCTION

With the advent of modern composites it has become important to have analytical solutions which can describe the elastic behavior of practical types of composite structures under load. As composites are orthotropic in behavior, an anisotropic formulation becomes imperative. Earlier works done for isotropic materials for a single bolt in an infinite plate by Bickley[1] and extensions of his analysis to treat finite boundaries.[2-4] The present paper treats a mixed-boundary value problem connected with the rigid bolt in a finite rectangular plate using the Lekhnitskii complex variable approach.[5] For plane problems of orthotropy, Lekhnitskii[5] has developed a solution in terms of analytic functions of complex variable theory along the lines of the well-known techniques of Muskhelishvili.[6]

The problem considered here is the problem of a rigid bolt in a finite rectangular sheet. The load is imposed by the bolt and its effect is represented as a known displacement on the hole of the same diameter as the bolt. The plate is kept in equilibrium by a uniform stress on one edge of the plate. Two types of boundary conditions have been considered at the lateral outer boundaries, one corresponding to a single bolt in a rectangular sheet and the other to a series of bolts equally spaced in a direction normal to the direction of the applied load. The boundary conditions are satisfied by a collocation method used by the author in Reference 7.

FORMULATION

Standard elements of a mechanical fastening system are shown in Figure 1. The essential part is an orthotropic plate with a circular cutout to accommodate a rigid circular bolt of the same radius. Force is applied to the plate by a translation displacement applied to the bolt, which causes a deformation on the circular hole whose pattern is known. A frictionless contact has been assumed. The plate is kept in equilibrium by a uniform tensile stress σ_A applied on the left edge of the plate in Figure 1. In practice such plates are fabricated of several layers, each having orthotropic properties and varying angles of inclination. The axis of symmetry in this particular problem is the x-axis.

In Figure 1 two types of joint configurations have been shown, a single pin and a periodic array of pins. Figure 1c shows an isolated pin well removed from the boundaries where an infinite plate solution will apply.*

*OPLINGER, D. W., and GANDHI, K. R. *Analytical Failure Studies in Mechanically Fastened Composite Plates.* Unpublished research.

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7. GANDHI, K. R. *Analysis of an Inclined Crack Centrally Placed in an Orthotropic Rectangular Plate.* J. Strain Analysis, v. 7, no. 3, 1972, p. 157-162.

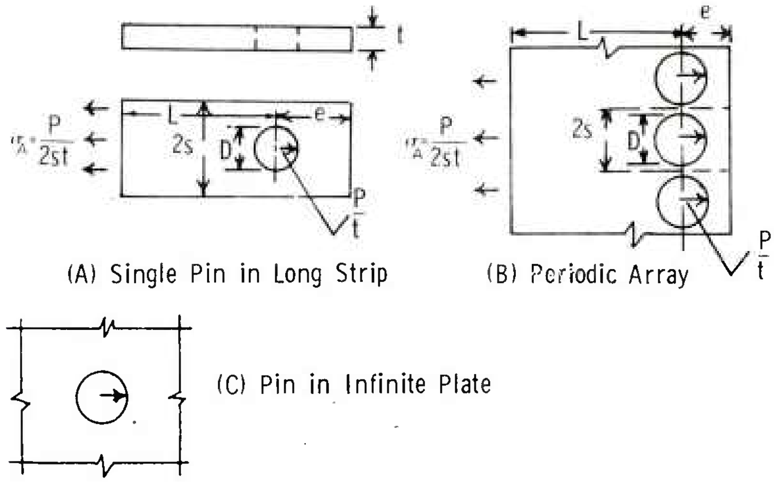

Figure 1. Mechanical joint configurations under consideration.

The problem has been treated as a plane problem ignoring the effect of out-of-plane and shear deformations. Mathematically, the problem has been formulated using the functions of complex variables. Adopting the conventional notation, the material properties can be described by the elastic constants E_1, E_2, ν_{12}, ν_{21}, and G_{12}. By symmetry of the constitutive matrix, $\nu_{12}/E_1 = \nu_{21}/E_2$.

The formulation of the analysis will now be briefly summarized. The Airy stress function for two-dimensional problems of rectilinear anisotropy can be written as

$$U(x,y) = \text{Re}\, U_1\{(z_1) + U_2(z_2)\}, \tag{1}$$

where U_1 and U_2 are analytic functions of the complex variables z_1 and z_2. The variables z_1 and z_2 correspond to

$$z_1 = x + s_1 y$$

$$z_2 = x + s_2 y, \tag{2}$$

where s_1 and s_2 are the complex roots of the characteristic equation

$$a_{11}\mu^4 - 2a_{16}\mu^3 + (2a_{12} + a_{66})\,\mu^2 - 2a_{26}\mu + a_{22} = 0. \tag{3}$$

Equation 3 defines the condition required for the Airy function U to satisfy the two-dimensional compatibility equation.

In Equation 3 the a_{ij} are the compliance coefficients occurring in the generalized Hooke's Law. Assuming unequal roots in Equation 3, we have

$$s_1 = \mu_1 = \alpha_1' + i\beta_1'$$

$$s_2 = \mu_2 = \alpha_2' \pm i\beta_2'$$

$$s_3 = \bar{s}_1 \quad s_4 = \bar{s}_2, \tag{4}$$

where α_j' and β_j' are real constants. Without loss of generality we assume that $\beta_1' > 0$ and $\beta_2' > 0$.

To simplify notation the new functions introduced are:

$$\phi_1\ (z_1) = dU_1/dz_1,\ \phi_2\ (z_2) = dU_2/dz_2. \tag{5}$$

Then, from the definition of the Airy stress function, the stresses can be written as

$$\sigma_x = 2\text{Re}\ \{s_1^2\phi_1'(z_1) + s_2^2\phi_2'(z_2)\}$$

$$\sigma_y = 2\text{Re}\ \{\phi_1'(z_1) + \phi_2'(z_2)\}$$

$$\tau_{xy} = -2\text{Re}\ \{s_1\phi_1'(z_1) + s_2\phi_2'(z_2)\}, \tag{6}$$

where, henceforth, primes will be used to denote differentiation with respect to the indicated arguments.

Another useful relation is the following expression for the force resultant as a function of arc length on an arbitrary curve:

$$(1+is_1)\phi_1(z_1) + (1+is_2)\phi_2(z_2) + (1+i\bar{s}_1)\overline{\phi_1(z_1)} + (1+i\bar{s}_2)\overline{\phi_2(z_2)}$$
$$= i\int^s(X_n+iY_n)ds = f_1(s) + if_2(s). \tag{7}$$

The displacements are expressed as

$$u = 2\text{Re}[p_1\phi_1+p_2\phi_2]$$

$$v = 2\text{Re}[q_1\phi_1+q_2\phi_2],$$

with p_k and q_k (k=1,2) given by

$$p_k = a_{11}s_k{}^2 + a_{12}-a_{16}s_k$$

$$q_k = a_{11}s_k{}^2 + a_{22}/s_k-a_{26}. \tag{8}$$

METHOD OF ANALYSIS

We now need to determine $\phi_1(z_1)$ and $\phi_2(z_2)$ so as to satisfy the stress and displacement boundary conditions. Since the circular boundary is transformed to an elliptical shape in the z_1 and z_2 planes, the first step consists of finding a simple mapping function which maps elliptical contours in the z_k plane on a unit circle in the plane of an auxiliary parameter $\zeta_k(z_k)$. The following accomplishes this step:

$$z_k = \omega(\zeta_k) = (R(1-is_k)/2)\zeta_k + (R/2)(1+is_k)(1/\zeta_k) \tag{9}$$

where R is the radius of the hole. The following are useful notations:

$$\phi_1(z_1) = \phi_1[\omega(\zeta_1)] = \phi_1(\zeta_1)$$

$$\phi_2(z_2) = \phi_2[\omega(\zeta_2)] = \phi_2(\zeta_2). \tag{10}$$

Thus

$$\phi_1'(z_1) = \phi_1'(\zeta_1)/\omega'(\zeta_1)$$

$$\phi_2'(z_2) = \phi_2'(\zeta_2)/\omega'(\zeta_2). \tag{11}$$

The transformed expression for the stresses can now be written as

$$\sigma_x = 2\mathrm{Re}\{(s_1^2\phi_I'(\zeta_1)/\omega'(\zeta_1)) + (s_2^2\phi_2'(\zeta_2)/\omega'(\zeta_2))\} \tag{12}$$

The resultant force, Equation 7, becomes

$$(1+is_1)\phi_1(\zeta_1) + (1+is_2)\phi_2(\zeta_2) + (1+i\bar{s}_1)\overline{\phi_1(\zeta_1)}$$
$$+ (1+i\bar{s}_2)\overline{\phi_2(\zeta_2)} = f_1(s) + if_2(s). \tag{13}$$

The boundary conditions involved in the analysis included specifications of normal and tangential stresses at the rectangular outer boundary together with mixed boundary conditions on the circular inner boundary. On the inner boundary the conditions are:

$r = R,\ 0 \le \theta \le \pi;\ \tau_{R\theta} = 0$ (frictionless case)

$\eta \le \theta \le \pi;\ \sigma_R = 0$

$0 \le \theta \le \eta;\ u_R = \delta \cos\theta.$

The second and third of these conditions imply that the contact angle is identified with η. With δ taken to be the rigid body displacement of the pin, the radial displacement u_R is given to a good approximation (neglecting higher order terms in δ/R) by $\delta \cos\theta$ in the contact region. The value of η is an unknown in the problem and was determined by a method described later.

On the outer boundary the boundary conditions are:

$x = e,\ 0 \le y \le s;\ \sigma_x = \tau_{xy} = 0$

$x = -L,\ 0 \le y \le s;\ \sigma_x = \sigma_A,\ \tau_{xy} = 0$

$-L \le x \le e,\ y = 0;\ v = \tau_{xy} = 0$

$-L \le x \le e,\ y = s;\ \tau_{xy} = 0$, (single pin) $\sigma_y = 0$

$-L \le x \le e,\ y = s;\ \tau_{xy} = 0$, (multipin) = constant.

This implies that the loaded end of the plate is located at $x = -L$. The third equation represents the assumption that the x-axis is a line of symmetry for the problem. In the last equation the single pin case corresponds to a free boundary at $y = s$, while in the multipin (i.e., periodic array) case $y = s$ represents a second line of symmetry. In the case of the contact angle, the value of v in the latter situation is an unknown. More precisely, it is required that the constant value of v at $y = s$ be adjusted so that the mean value of σ_y along this boundary is zero. In the approach used here, two trial values for v were assumed and the total vertical force f_y on the boundary was calculated as part of the solution. Since f_y is a linear function of v, a straight line relation was thereby established which determined the value of v required to set f_y (and therefore the mean σ_y) equal to zero. In the infinite plate case, of course, the outer boundary conditions do not enter. Correspondingly, the positive powers in the Laurent series, Equation 14, are eliminated. Other simplifications which were used to advantage in the infinite plate case are described in Equation 13.

The boundary condition equations contain the unknown stress functions $\phi_1(\zeta_1)$ and $\phi_2(\zeta_2)$, which are the analytic functions of their arguments in a doubly connected region. Since we are dealing with this type of region, a Laurent series of the following form has been selected:

$$\phi_1(\zeta_1) = A_1 \log \zeta_1 + \sum_{n=-\infty}^{+\infty} (\alpha_n + i\beta_n)\zeta_1^n$$

$$\phi_2(\zeta_2) = A_2 \log \zeta_2 + \sum_{n=-\infty}^{+\infty} (\gamma_n + i\delta_n)\zeta_2^n .$$

A further simplification can be made knowing the symmetries in the elastic properties of composite materials. The following major kinds are considered here. Both of them ensure that x-axis is the axis of symmetry:

a. when the characteristic roots s_1 and s_2 are purely imaginary, i.e., $\alpha_1' = \alpha_2' = 0$; and

b. when the roots are related by the equation $s_2 = -\bar{s}_1$.

The above symmetries are obtained depending upon the manner in which composite materials are put together. Most of the composite designs would fall into one or the other category. In the symmetry of type a it can be shown that under that special condition, $\alpha_n = \gamma_n = 0$, while in the symmetry of type b, $\delta_n = -\gamma_n$ and $\beta_n = \delta_n$. Further, the complex coefficients of the log terms A_1 and A_2 are determined by imposing the single value condition on f_1 and f_2 and the displacements u and v. As derived by Lekhnitskii,[4] the following equations are relevant. The values of A_1 and A_2 depend entirely on the magnitude of the pin load $P = 2s\sigma_A$.

$$A_1 + A_2 - \bar{A}_1 - \bar{A}_2 = 0$$

$$s_1A_1 + s_2A_2 - \bar{s}_1\bar{A}_1 - \bar{s}_2\bar{A}_2 = -P/2\pi i$$

$$P_1A_1 + P_2A_2 - \bar{P}_1\bar{A}_1 - \bar{P}_2\bar{A}_2 = 0$$

$$q_1A_1 + q_2A_2 - \bar{q}_1\bar{A}_1 - \bar{q}_2\bar{A}_2 = 0$$

Now the problem simplifies to selecting the unknowns α_n, β_n, γ_n, and δ_n in Equation 14 so that the external boundary conditions are satisfied with sufficient accuracy A least-squares collocation procedure[7] was used and found satisfactory. It consists of the following steps.

1. Truncation of the infinite expansion in Equation 14 to a suitable finite number of positive and negative powers.

2. Selection of points or stations around the external boundary in a fairly uniform manner and writing force equations at each location. The number of such equations was always arranged to provide more than twice the number of unknowns.

3. Solutions of unknowns in such a manner that the equations in step 2 are satisfied in a least-squares sense.

ANGLE OF CONTACT

As in any contact problem, the determination of the region of contact is a part of the solution. As the contact length depends on the load, the problem strictly speaking is nonlinear. In our problem we employed a simple iterative procedure which, within a few trials, succeeded in arriving at a close approximation to the true contact angle η starting with an assumed value close to 90°. The method relies on the obvious physical fact that the radial stress σ_r in the contact region is always compressive and approaches zero at the end of contact in a well-behaved manner. Therefore, the procedure consists of starting with a value of η denoted by η_1, solving the problem, and examining the sign of σ_r at $\theta = \eta_1$. If the sign of σ_r is compressive, it can be concluded that the true value of η is greater than the assumed value η_1. In either case a solution is obtained with a new trial value of η, say η_2. In all the cases tried, the two trial values, η_1 and η_2, were sufficient to give a relatively accurate estimate of the true value of η. This approach amounts to a procedure similar to the Newton Raphson method for nonlinear algebraic equations. In the region close to the true value of η, the assumed contact angle plotted against σ_r at $\theta = \eta$ gives a nearly linear plot. The iterative method converges quite rapidly.

NUMERICAL RESULTS

On the basis of the foregoing analysis, a computer code was developed to determine the coefficients of the stress functions. The procedure consisted of selecting discrete stations around the boundary at which the prescribed boundary conditions were specified. At each of these stations two equations expressing the pertinent boundary conditions were written. The stations were roughly equally spaced and their numbers so arranged that they gave a large number of equations, more than twice the number of known coefficients in the stress function. The resulting redundant system was solved by the least-squares collaction procedure.[7] To investigate the accuracy of the solution, some arbitrary points on the boundary were examined for boundary conditions. For the range of the results presented encompassing different material and geometrical configurations, a fairly high order of accuracy was obtained even with the truncation of the series to $-10 \leq n \leq +10$.

The practical motivation in carrying out this analysis was to develop a tool to predict the complicated behavior of mechanical joints of laminated composites typical of aircraft structural applications. The elastic properties of layers are given in Table 1. The fabrication of laminates for a joint is usually done by stacking layers at different angles of inclination to the axis of the joint. The elastic properties of such a joint were estimated by doing the usual property transformations.[8] A simple laminate theory was used to estimate the average elastic properties of the laminate. This simple theory ignores the out-of-plane phenomena such as interlaminar shear stresses, etc.

Table 1. ASSUMED MECHANICAL CONSTANTS OF FIBER-REINFORCED MATERIALS

Elastic Constants	Graphite Epoxy	Glass Epoxy
E_L, 10^6 psi (GN/m^2)	25.0 (172)	7.00 (48.2)
E_T, 10^6 psi (GN/m^2)	1.0 (6.89)	2.50 (17.2)
G_{LT}, 10^6 psi (GN/m^2)	0.7 (4.82)	1.20 (8.26)
ν_{LT}	0.2	0.25

Figure 2 shows the general distribution of stresses on the fastener hole periphery. In some cases the distribution of σ_r has the appearance of "half cosine" distribution. As is true in the anisotropic solutions, sizable variations can take place in any direction depending upon the configuration of material and geometries. An example of interest is given in Figure 2 for the curve marked with squares, where e = 1.5 and the material is 0° glass epoxy. In Figure 2 where e/R = 1.5, the method of solution places certain limitations on geometry for its successful results. One limitation is due to the convergence of a Laurent series for any awkward shape. For example, when either e/r or s/r is close to unity, the boundary conditions are satisfied in a poor way. One more observation suggests that for large values of e and s the solution gradually merges into the solution for infinite plate. The method of solution was relatively faster in convergence for periodic arrays than for the single pin case with the same number of terms needed for a given level of accuracy in the satisfaction of the boundary conditions. The bulk of the results tried were for the perfect fit case, i.e., pin radius equals the hole radius. This method was demonstrated to be suitable for

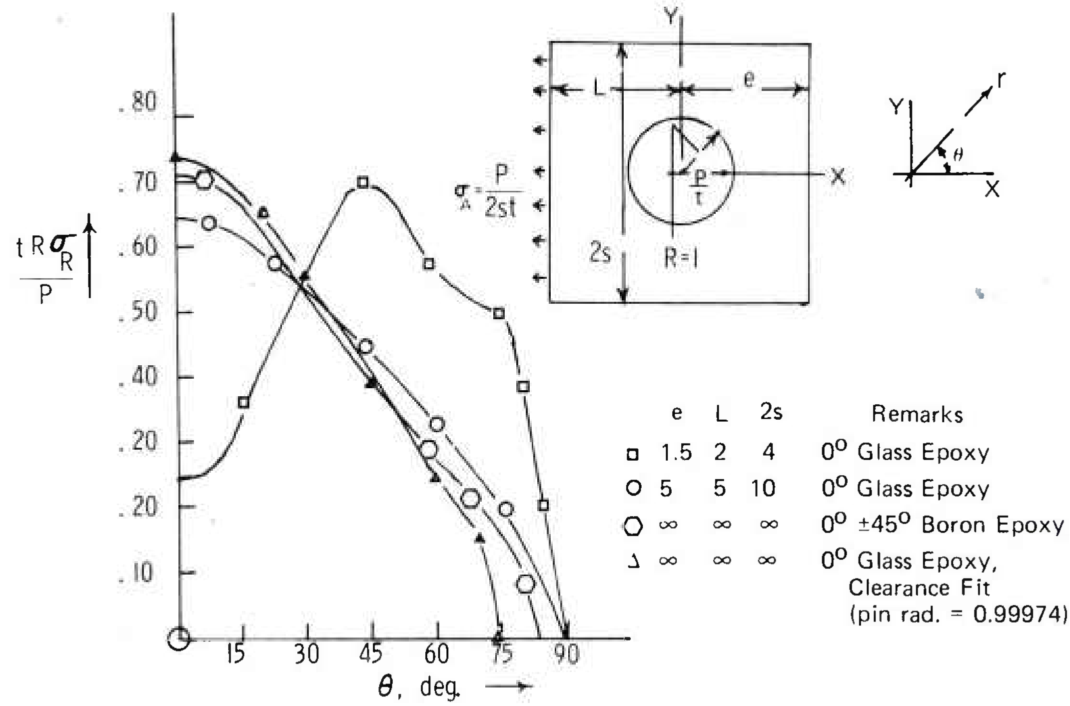

Figure 2. Radial stress distribution around fastener hole (D = 2).

8. HALPIN, J., ASHTON, J., and PETIT, P. *Primer on Composite Materials Analysis.* Technomic Publishing Co., 1969.

clearance fit cases, particularly when the difference between pin and hole radius was small. One such example is shown in Figure 2 labeled with triangles, applying to a pin radius of 0.99974.

It is customary to compute stress concentration factors, which in turn help in estimating the stress peaks in comparison with the mean value of stresses. Now we shall define the following parameters which are of practical interest to the designers of composite materials.

Mean Net Section Tension, $\bar{\sigma}_{NS} = P/2(s-D/2)t$

Mean Bearing Stress, $\bar{\sigma}_B = P/2Dt$

Mean Shearout Stress, $\bar{\tau}_{so} = P/2et$

Further, these mean stresses can be divided by the applied load σ_A.

$$k_{NS} = \bar{\sigma}_{NS}/\sigma_A = s/(s-D/2)$$

$$k_B = \bar{\sigma}_B/\sigma_A = 2s/D$$

$$k_{so} = \bar{\tau}_{so}/\sigma_A = s/e.$$

The following ratios defined as stress concentration factors are the ratio of the peak stresses to the corresponding mean values.

$$K'_{NS} = \sigma_\theta)_{\theta = 90,\ r = R}/\sigma_A$$

$$K'_B = \sigma_B)_{max}/\sigma_A$$

$$K'_{so} = \tau_{xy})_{max}/\sigma_A.$$

The quantity $\sigma_B)_{max}$ is defined as the compressive axial stress, i.e., maximum compressive value of σ_n along the circular hole ($r = R$) in the contact region. Similarly, $\tau_{xy})_{max}$ is the maximum shear at $r = R$. It was observed that the maximum τ_{xy} occurs between polar angles of about 40° to 70° in the region of contact.

Finally, the ratios of $\sigma_\theta)_{\theta = 90°,\ r = R}$ $\sigma_B)_{max}$, and $\tau_{xy})_{max}$ to σ_A are of interest. These are denoted as

$$K_{NS} = \sigma_\theta)_{\theta = 90,\ r = R}/\bar{\sigma}_{NS}$$

$$K_B = \sigma_B)_{max}/\bar{\sigma}_B$$

$$K_{so} = \tau_{xy})_{max}/\bar{\tau}_{so}.$$

The following relations are apparent:

$$K'_{NS} = k_{NS}K_{NS}$$

$$K'_B = k_B K_B$$

$$K'_{so} = k_{so} K_{so}.$$

Solutions were computed for various geometries. The important parameters for observation are e/D and s/D. The material was high-modulus graphite and the fiber orientation was $0_2 \pm 45$. The relevant elastic properties are shown in Table 1. The boundary condition was for the multiple pin case. The effect of varying the geometry is shown in Figures 3 and 4. Figure 3 shows the effect of s/D on the stress concentration factors K_{NS}, K_B, and K_{so}. These curves stop at the indicated dotted lines because s cannot be less than D/2. Figure 3 also indicates the linear trends predicted by the infinite plate solution which shows the values of s/D for which the finite plate solution merges with the infinite plane solution. Figure 4 shows the variation of the stress ratios with normalized edge distance (e/d) corresponding to s/D = 1. It is interesting to note that the curves fast approach a constant level for e/D ≥ 2.

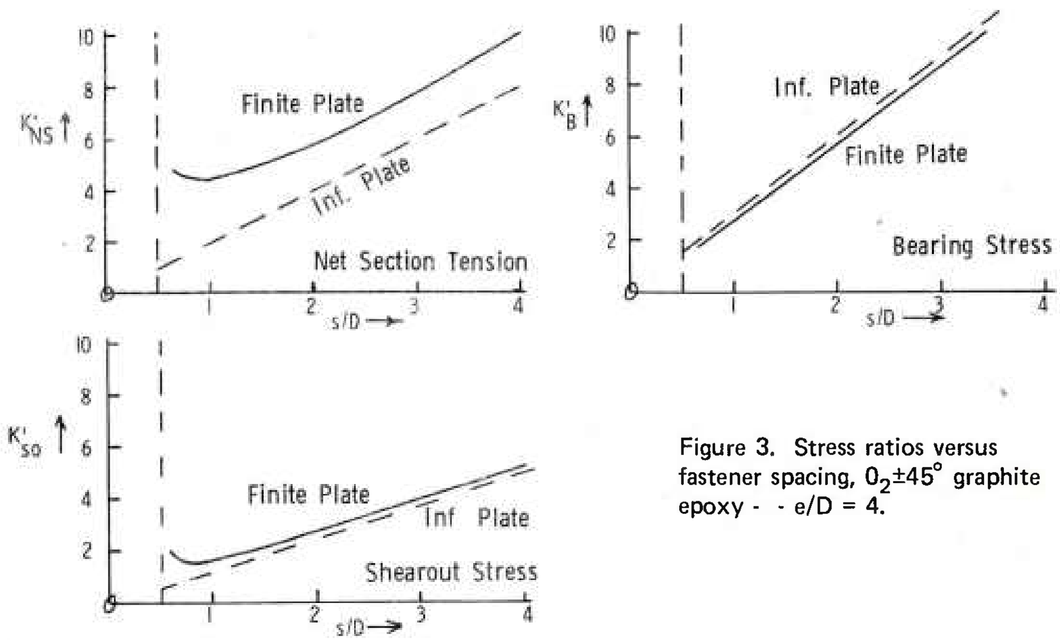

Figure 3. Stress ratios versus fastener spacing, $0_2 \pm 45°$ graphite epoxy - - e/D = 4.

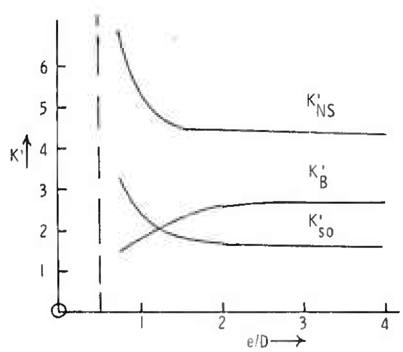

Figure 4. Stress ratios versus edge distance, $0_2 \pm 45°$ graphite epoxy - - s/D = 1.

Figure 5 shows some variations of σ_x at various cross sections. We have observed that at a distance equal to twice the diameter away from the hole the stresses become nearly uniform.

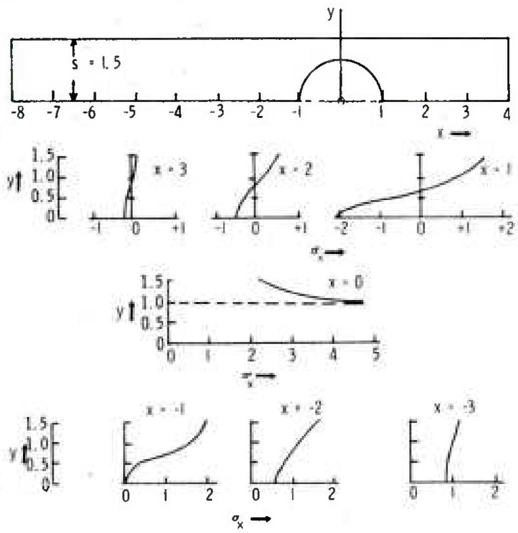

Figure 5. Variation of axial stress profile along x-axis, $0_2 \pm 45°$ graphite epoxy, e/D = 2, s/D = 0.75, L/D = 4 (multipin).

CONCLUSIONS

The results show the complicated nature of the strain distributions in composite joints. The Laurent series approach brings out the sensitive points in the analytic model such as the normal stress distribution around the hole as well as variation of the contact angle. The normal stress σ_r is often shown to be far from the usual cosine distribution. The angle of contact is peculiar to the elastic properties of the material at the aspect ratio in consideration. Theoretically, it is also dependent upon the load level, but for the elastic properties of the composites considered and the magnitude of the load the joint would be called upon to withstand, the contact angle can reasonably be considered constant. The method gives good solutions in the range of geometries encountered in practical engineering design situations. The solution scheme breaks down when the geometry is too unsymmetrical about the center of the hole. When the geometry enlarges about the center fairly uniformly in all directions, the solution merges with the solution for infinite plate.

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CONTACT PROBLEM FOR AN ORTHOTROPIC PLATE
WITH A CIRCULAR HOLE LOAOEO BY A BOLT -
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